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LETTER TO THE EDITOR

Confluent singularities of the renormalised coupling constant at $D=3$

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Abstract. We give estimates, from the massive field theory at $d=3$, of the universal ratio a_g^+/a_χ^+, where a_g^+ and a_χ^+ are the first confluent correction amplitudes of, respectively, the renormalised coupling constant g and the susceptibility χ above the critical temperature. We also give an analytical expression (numerically determined) for g as a function of the temperature which reproduces the crossover between the values $g\simeq 0$ and $g\simeq g^*$ (g^* is the fixed point value of g). This means that the infinite series of confluent corrections to g^*, generated within the Φ^4 model, are resummed.

In order to test the hyperscaling hypothesis, it is convenient to study (Baker 1977) the dimensionless renormalised coupling constant g defined as

$$g=-\xi^{-d}(\partial^2\chi/\partial H^2)/\chi^2 \tag{1}$$

where ξ and χ are, respectively, the correlation length and the susceptibility; H is the magnetic field. g behaves as $(T-T_c)^x$ where T is the temperature, T_c its critical value; $x=\omega^*\nu$, ν is the critical exponent of ξ and ω^* the anomalous dimension of the vacuum which vanishes if hyperscaling holds. In that case, which corresponds to the field theory (FT) framework, g tends to a finite g^* (the fixed point value of g).

The hyperscaling hypothesis has been much discussed (see, for example, Nickel 1982) owing to a possible failure suggested by high temperature series expansion (HTSE) analysis which led to $\omega^*>0$ (Baker 1977). However, it has been realised that carefully taking into account confluent singularities could be essential in such an analysis (Nickel 1982).

For $T\simeq T_c$, most of the quantities depending on T have a general asymptotic critical behaviour which reads as follows:

$$f^+(t)\simeq A_f^+|t|^{-\lambda_f}(1+a_f^+|t|^\Delta+\ldots) \tag{2}$$

where t is proportional to $T-T_c$ for $T\simeq T_c$, λ_f and Δ (A_f^+ and a_f^+) are the leading and first corrections to scaling critical exponents (amplitudes). The superscript + refers to the disordered phase. It is well known that the various ratios a_f^+/a_h^+ are universal (f and h denote two different quantities). Some of these ratios have been estimated from FT techniques in an ε-expansion framework (Aharony and Ahlers 1980, Chang and Houghton 1980, Nicoll and Albright 1985) and in the massive theory directly at $d=3$ (Bagnuls and Bervillier 1981, 1985a). As for a_g^+, only ε expansion up to first order has been carried out (Chang and Rehr 1983) leading to a poor numerical accuracy of the value of a_g^+/a_χ^+.

In this letter we give estimates of a_g^+/a_χ^+ at $d=3$ from the massive field theory using the long perturbative series of Nickel *et al* (1977). Owing to the greater numerical accuracy that we obtain using the sophisticated resummation method of perturbative series (Le Guillou and Zinn-Justin 1980), we think that we supply some useful information on the numerical verification of the validity of hyperscaling.

We first obtain estimates of a_g^+/a_χ^+ by direct resummation of the series of the various terms which appear in equation (19)† of Chang and Rehr (1983). We rewrite this equation in the following form:

$$\frac{a_g^+}{a_\chi^+}=\frac{\Delta(\Delta+1)}{g^*\nu[\gamma_3^{(1)}+\gamma(\gamma_4^{(1)}-\gamma_3^{(1)})]} \tag{3}$$

in which γ is the critical exponent of χ, $\gamma_3^{(1)}$ and $\gamma_4^{(1)}$ are defined in equations (2.14) and (2.15) of Bagnuls and Bervillier (1981). The values so obtained are displayed in table 2.

We also present here another formulation of the confluent singularities effect which may be more useful for a comparison with experimental data (Dohm 1984) and HTSE analysis. We give an analytic expression for $g(t)$ whose validity, within the Φ^4 model, goes far away from criticality.

The main ingredients for obtaining $g(t)$ have already been described in great detail in Bagnuls and Bervillier (1985a). We shall restrict ourselves here to a brief presentation of the numerical study of $g(t)$.

This function naturally arises, in the framework of FT, through its inverse which is expressed in terms of the renormalisation functions $Z_i(g)$ $(i=1\text{-}3)$, calculated up to sixth loop order (Nickel *et al* 1977), as follows (Bagnuls *et al* 1984, Bagnuls and Bervillier 1985a):

$$\tilde{t}(g)=-\int_g^{g^*}\mathrm{d}x\left[Z_2(x)\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{[Z_3(x)]^3}{[xZ_1(x)]^2}\right)\right] \tag{4}$$

in which $\tilde{t}$ is a dimensionless scaling field proportional to $(T-T_c)/T_c$. The bound g^* (the limit of g when $T\to T_c$) is the non-trivial zero of the Wilson function which is also expressed through Z_i. As shown by Symanzik (1973) (see also Bagnuls and Bervillier 1985a), $\tilde{t}(g)$ has a non-perturbative expression. In order to avoid logarithmic singularities (as encountered in Bagnuls and Bervillier 1983), we integrate after having resummed the power series of Z_i. This has been done numerically at various discretised values of g between 0 and g^*. The evolution of g in terms of $\tilde{t}$ is shown in figure 1(a) (dots). We then propose an analytic expression of $g(\tilde{t})$ for $n=1$, 2 and 3 (n is the spin dimensionality) which reproduces, with a relative error of less than 10^{-4}, the behaviour of $g(\tilde{t})$ for $\tilde{t}\leqslant 10^{-2}$. The form chosen for $g(\tilde{t})$ is

$$g(\tilde{t})=g^*[1+X_2\tilde{t}^{\Delta}]^{X_3}[1+X_4\tilde{t}^{\Delta}]^{X_5} \tag{5}$$

in which the X_i are simple adjusted numbers and are given in table 1. We perform our numerical integrations twice according to the cases $g^*_{\max}$ and $g^*_{\min}$ which correspond to the upper and lower values of the fixed point determined by an error analysis of the resummation method for the series.

† In this equation g^* appears, by error, on the numerator. We re-establish the correct dependence in equation (3) of the present letter.

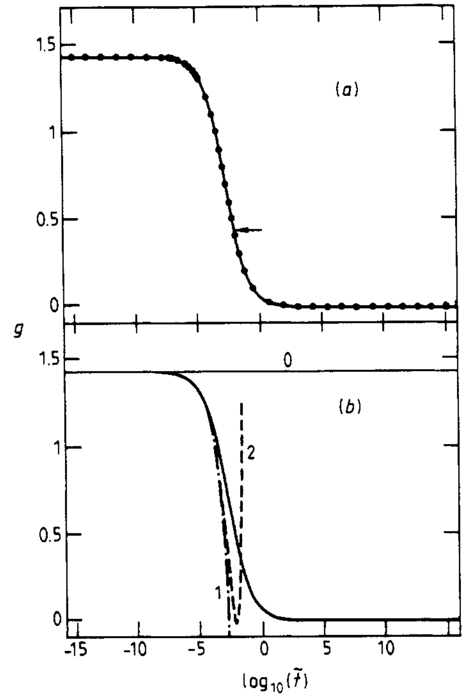

Figure 1. (a) Illustration of the quality of the adjustment of the parameters X_i ($i=2$–5) of equation (5) (full curve) to the discretised (dots) evolution of $g(\tilde{t})$ primarily obtained from numerical study of equation (4). The adjustment is made up to $\tilde{t}=10^{-2}$ (indicated by the arrow) with a relative error of less than 10^{-4}. One sees that the points corresponding to $\tilde{t}>10^{-2}$ are also well reproduced although not fitted. (b) Illustration of the range of the influence of the first (chain curve) (1) and second (2) confluent corrections to g^*. The pure scaling law behaviour would correspond to the straight horizontal line (0). One observes that in the range $\tilde{t}\leq 10^{-2}$ much more than one confluent correction is relevant. This supports the correctness of our determination of a_g^+ from equation (5) and table 1.

Table 1. Numerical values of the parameters X_i ($i=2$–5) obtained by adjustment of the function $g(\tilde{t})$ (equation (5)) to its discretised evolution given by the study of equation (4) for $n=1$–3. The two sets of values displayed (two successive lines) correspond to the bounds max (upper line) and min associated with the upper and lower values of g^* at which the Wilson function vanishes. This yields an indication of the numerical accuracy of the work. The values of g^* and Δ can be compared to the estimates obtained by Le Guillou and Zinn-Justin (1980). For a more complete discussion on the error estimates in this work see the text and Bagnuls and Bervillier (1985a).

n	g^*	Δ	X_2	X_3	X_4	X_5
1	1.420 215	0.491 25	29.0571	−0.454 744	16.897 3	−0.579 649
	1.410 942	0.500 31	29.6238	−0.694 946	11.857 5	−0.331 626
2	1.408 862	0.520 10	30.1399	−0.640 906	13.992 1	−0.334 648
	1.401 516	0.5274	32.4596	−0.662 867	11.997 3	−0.306 613
3	1.392 477	0.5498	32.9598	−0.673 133	11.394 4	−0.252 812
	1.390 814	0.5504	32.3769	−0.711 793	9.170 41	−0.224 708

From equation (5) we obtain

$$a_g^+ = X_2X_3 + X_4X_5 \tag{6}$$

which, combined with previous results for a_χ^+ (Bagnuls and Bervillier 1985a), leads to the estimates for a_g^+/a_χ^+ displayed in table 2. They are in agreement with results obtained with equation (3) but with smaller error bars. This error minimisation has two different origins. First, only one resummation method is used here (by contrast with the work of Le Guillou and Zinn-Justin (1980)), and second the error correlations are better taken into account (see Bagnuls and Bervillier 1985a).

Table 2. Numerical values of the ratio a_g^+/a_χ^+ for $n = 1$-3 obtained from equation (6), table 1 and Bagnuls and Bervillier (1985a) (second column), equation (3) and Bagnuls and Bervillier (1981) (third column), ε expansion (Chang and Rehr 1983) (fourth column), HTSE analysis (George and Rehr 1985) (fifth column). One can observe that the non-asymptotic formulation (equation (6)) gives values in agreement with those obtained by direct estimates from equation (3) with, however, reduced error bars (see text). The agreement with HTSE is better than that obtained from ε expansion which cannot give accurate information when limited to small orders.

In the sixth and seventh columns, we give the estimates of $\gamma_3^{(1)}$ and $\gamma_4^{(1)}$ as they were determined and used in Bagnuls and Bervillier (1981).

	a_g^+/a_χ^+					
n	From table 1	From equation (3)	ε expansion	HTSE	$\gamma_3^{(1)}$	$\gamma_4^{(1)}-\gamma_3^{(1)}$
1	−2.848 (64)	−2.8 (4)	−2.22	−3.2 (6)	0.054 (14)	−0.284 (11)
2	−2.084 (54)	−2.03 (25)	−1.75	—	0.049 (17)	−0.353 (11)
3	−1.652 (44)	−1.55 (24)	−1.49	—	0.056 (36)	−0.445 (20)

Beyond the knowledge of a_g^+ through a universal ratio, the function $g(\tilde{t})$ that we obtain may have an interest by itself in a comparison with experiments and HTSE analysis. Although the model has a validity limited to values of $T-T_c$ for which higher corrections than the first one are negligible, the knowledge of non-asymptotic critical behaviour within this model is very interesting because of the information then obtained on the true convergence of the Wegner expansion (Wegner 1972). As is shown in figure 1(*b*), one clearly sees the range of influence of the first two confluent corrections (within the Φ^4 model). A complete discussion of the use of equation (5) for concrete comparison (in particular the introduction of adjustable parameters) is given in Bagnuls and Bervillier (1984, 1985a, b).

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